Uncertainty Analysis Review

Reviving
THE *river* OF *grass*

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sfwmd.gov/riverofgrass

PRESENTATION HIGHLIGHTS

- Introduction to SFWMD including CERP
- SFWMD primary models
- Previous workshops on UA
- Uncertainty Analysis and Sensitivity Analysis Basic definition
- Sources and measures of uncertainty
- Uncertainty Analysis techniques
- Application to NSRSM to demonstrate the following:
  - Local sensitivity analysis
  - Global sensitivity analysis
  - Uncertainty Analysis techniques
  - Global Sensitivity Analysis
- Conclusion and lessons learned

Florida's Five Water Management Districts

Northwest Florida Water Management District

Suwannee River Water Management District

St. Johns River Water Management District

Southwest Florida Water Management District

South Florida Water Management District

SFWMD Mission

To manage and protect water resources of the region by balancing and improving water quality, flood control, natural systems and water supply

Regions

- Coastal Watersheds & Estuaries
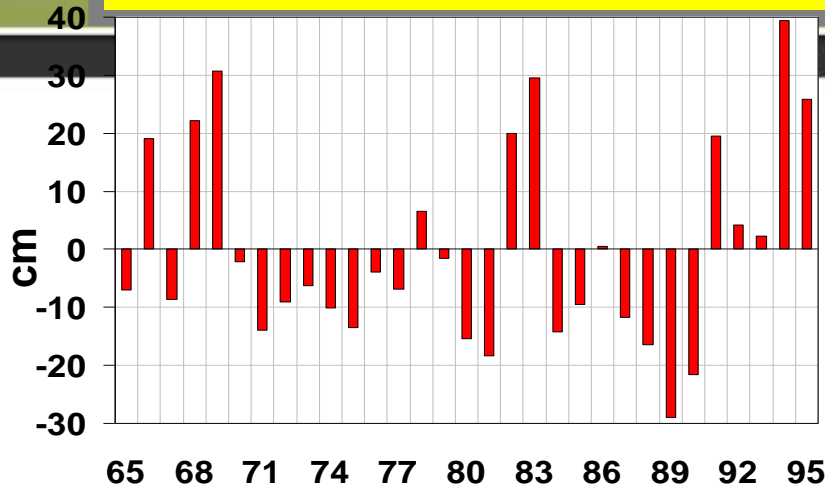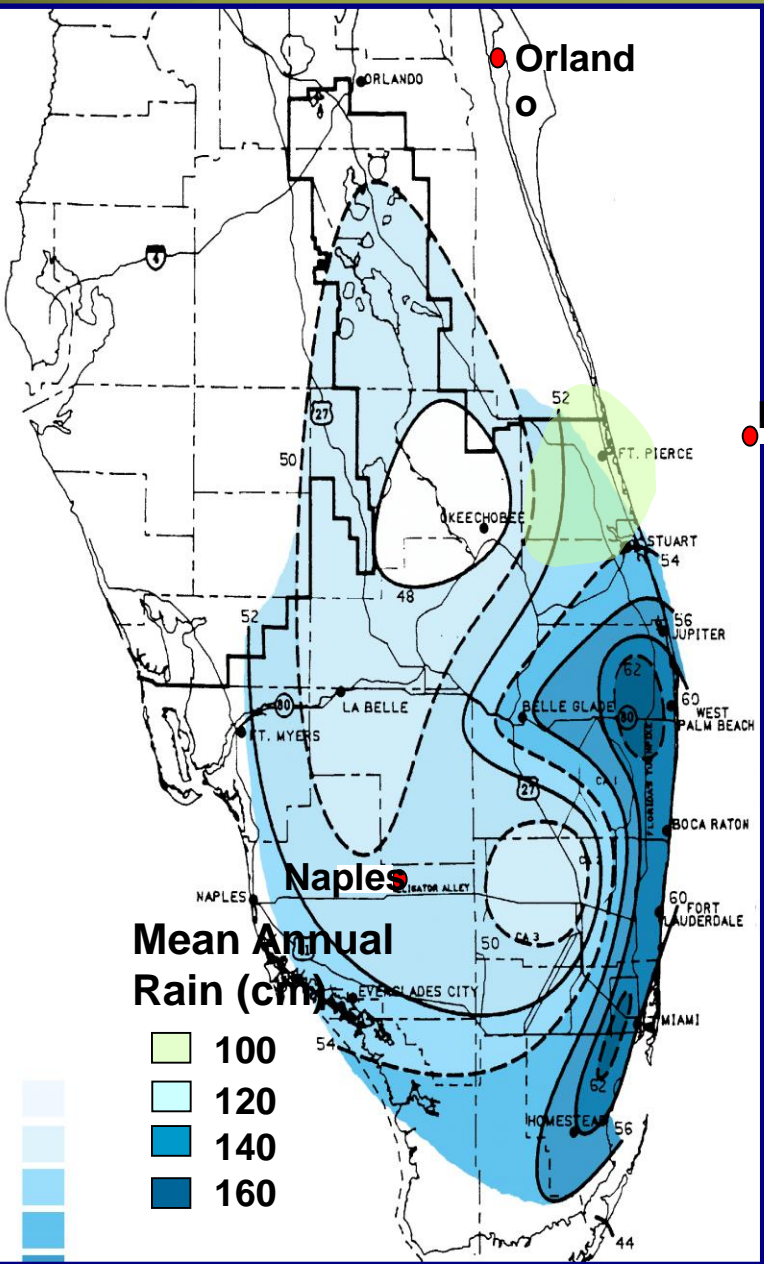- Everglades
- Kissimmee
- Lake Okeechobee

Objectives

- Mange Drought & Floods
- Provide Water Supply
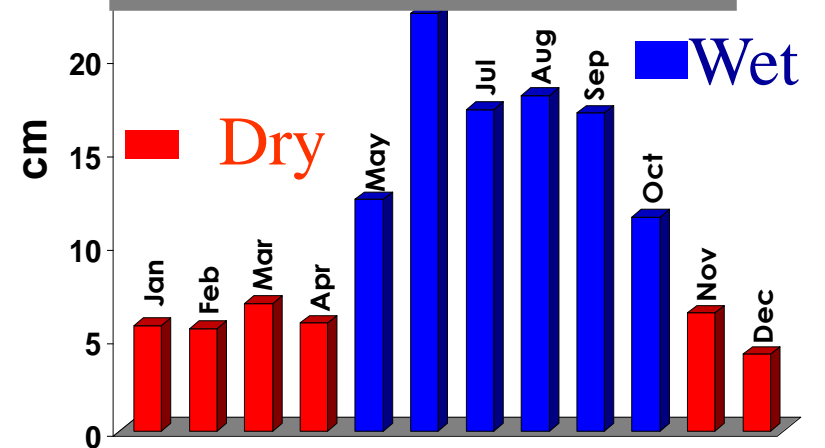- Protect & Restore Ecosystems
- Prepare for Emergencies

Functions

- Science, Planning, Engineering, & Construction
- Land Management
- Operation & Maintenance
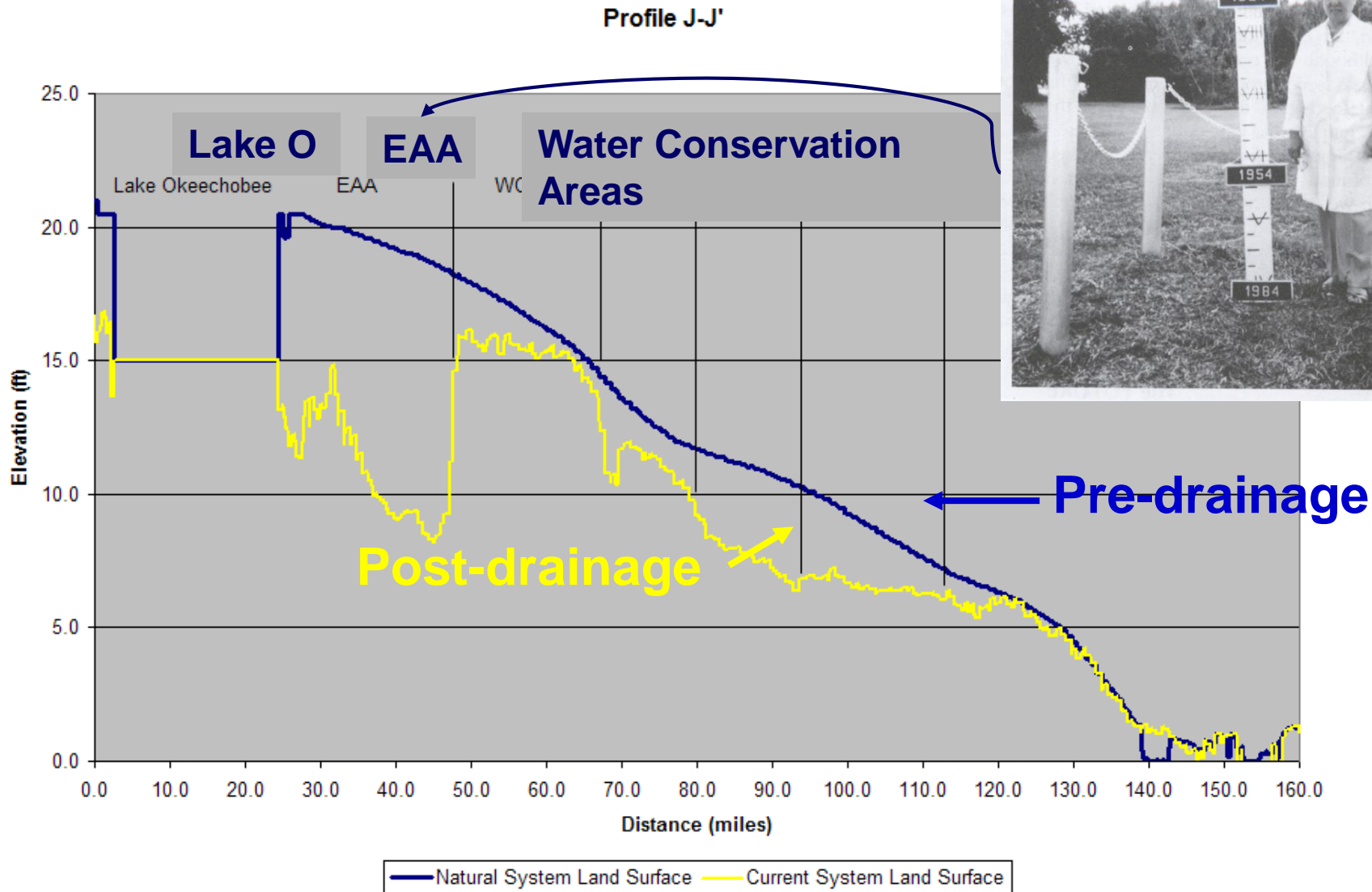- Regulation
- Water Supply Development

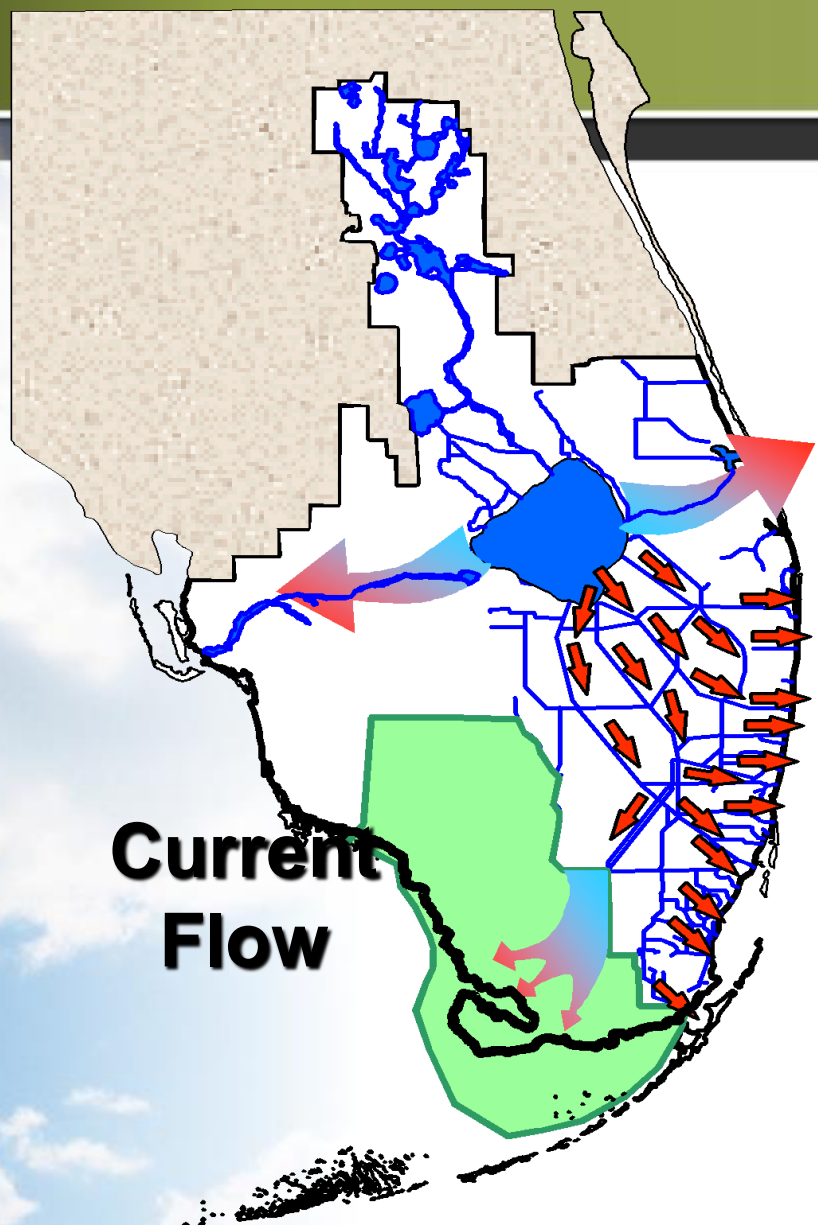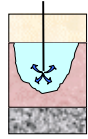**Rainfall Deviations from Mean of 133 cm**

Monthly Distribution

Wet

Dry

Mean Annual Rain (cm)
- 100
- 120
- 140
- 160

Orlando

Ft. Pierce

West Palm Beach

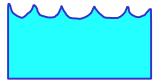Miami

Naples

Soil Subsidence

Current Flow

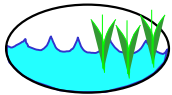# Our Ecosystem has been altered dramatically

CERP Components

Aquifer Storage & Recovery

Surface Water Storage Reservoir

Stormwater Treatment Areas (STAs)
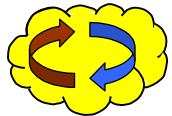
Reuse Wastewater

Seepage Management

Removing Barriers to Sheetflow

Operational Changes

Orlando

Kissimmee River

Caloosahatchee River

Lake Okeechobee

Ft. Myers

WCA

Big Cypress National Preserve

Everglades National Park

Florida Bay

Florida Keys

Biscayne Bay
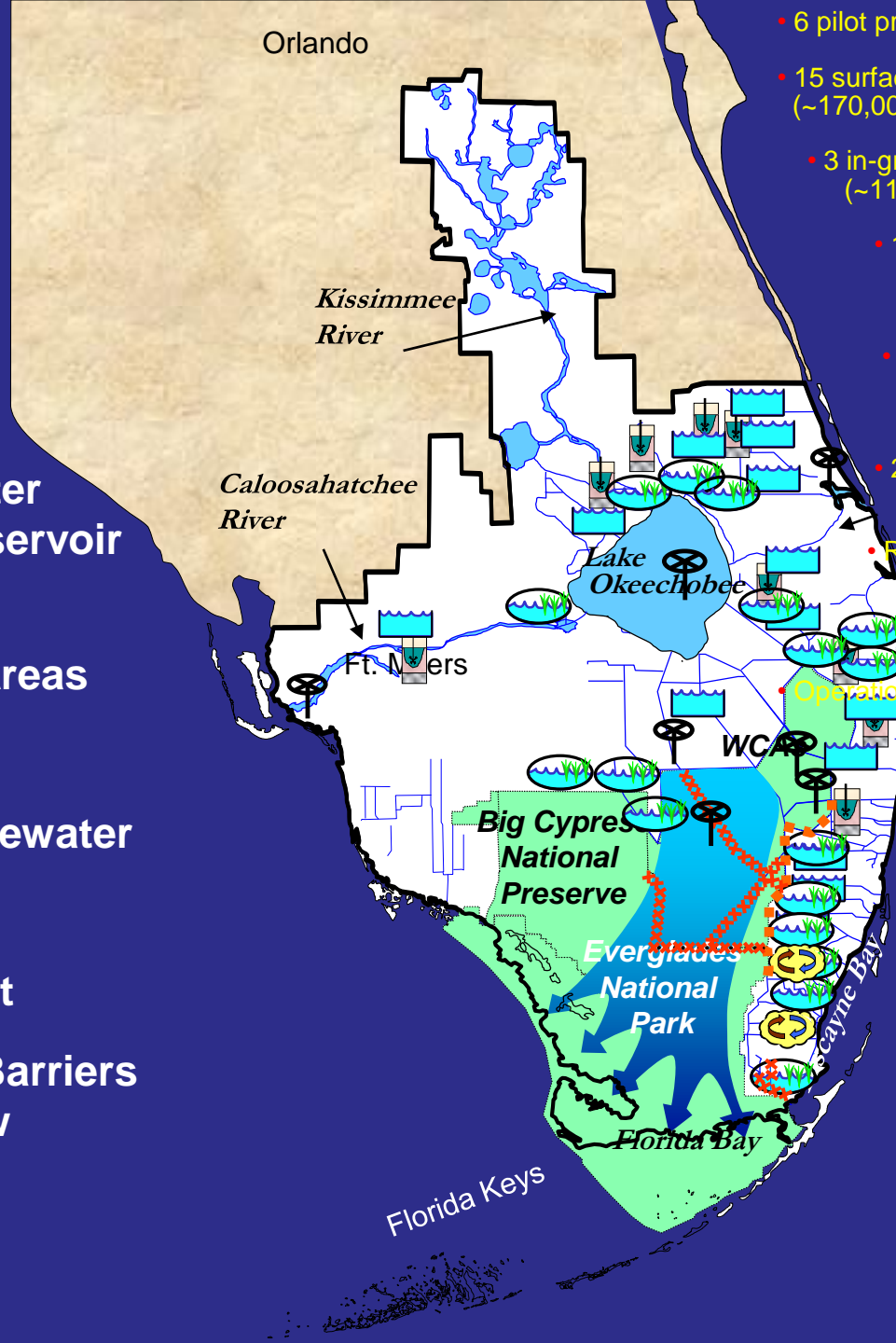
- 6 pilot projects
- 15 surface storage areas (~170,000 acres)
- 3 in-ground reservoirs (~11,000 acres)
- 19 stormwater treatment areas (~36,000 acres)
- 330 aquifer storage and recovery wells
- 2 wastewater reuse plants
- Removal of over 240 miles of canals, levees and structures
- Operational changes

Major C&SF Project Components

- River Channelization

- Herbert Hoover Dike

- Water Conservation Areas

- Protective Levees

  - Everglades Agricultural Area

  - Lower East Coast

- Drainage Network

  - Salinity Structures

What is a MODEL?

Input data at limited sites in space/time

System's parameters & BC measured at limited points

Mathematical representation of the system processes and Numerical implementation

Management Decisions

Prediction
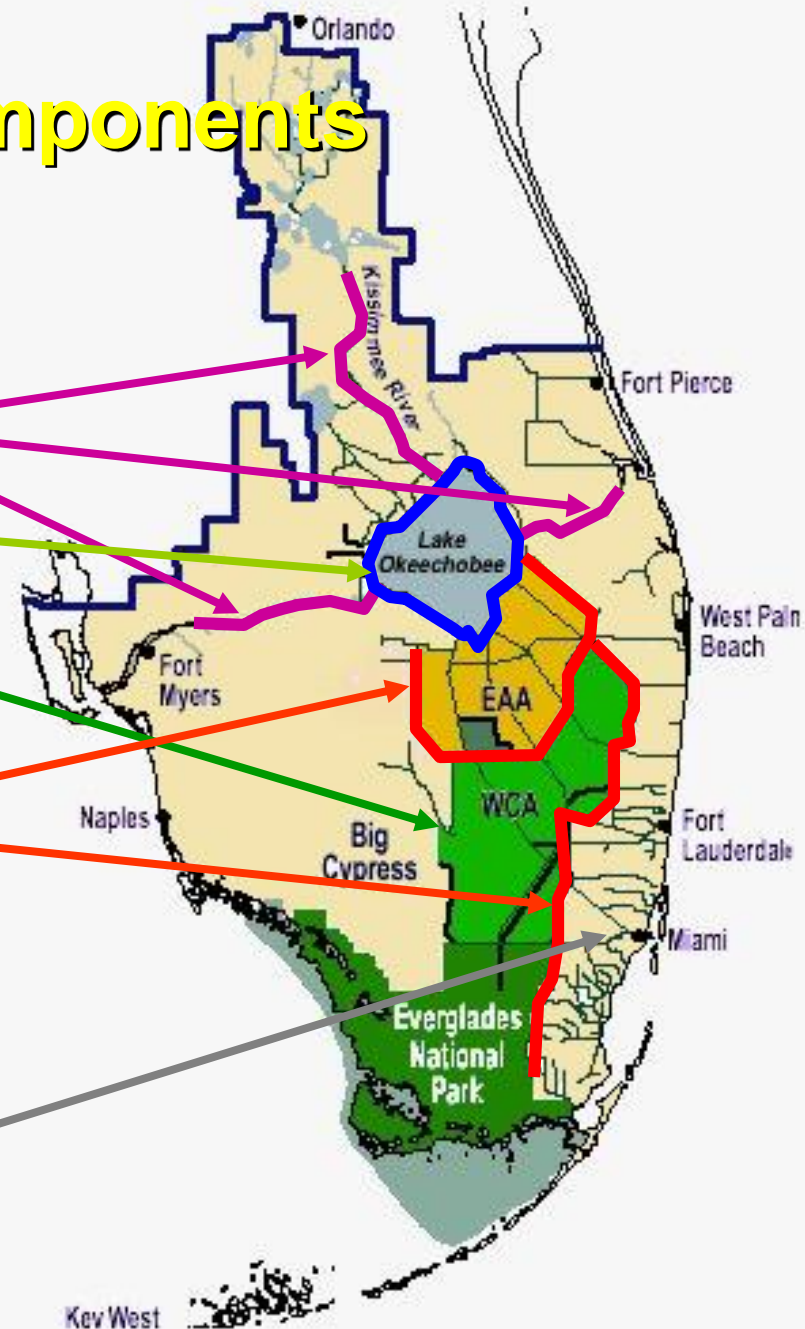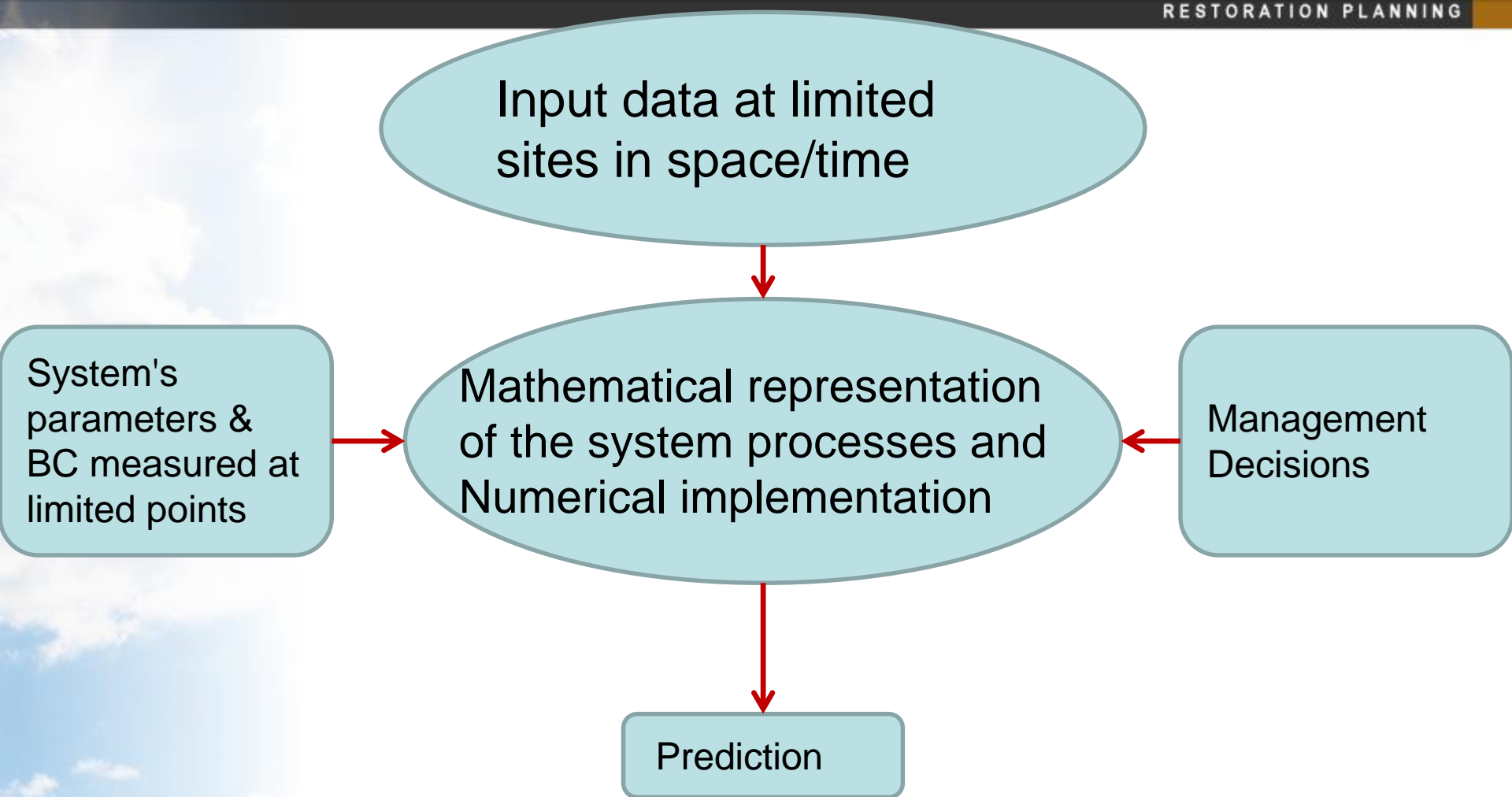
South Florida Water Management Model (SFWMM)

- **Integrated surface water groundwater model**
- **Regional-scale 3.2 x 3.2 km, daily time step**
- **Major components of hydrologic cycle**
- **Overland and groundwater flow**
- **Canal and levee seepage**
- **Operations of C&SF system**
- **Water shortage policies**
- **Extensive performance measures**
- **Provides input and boundary conditions for other models**
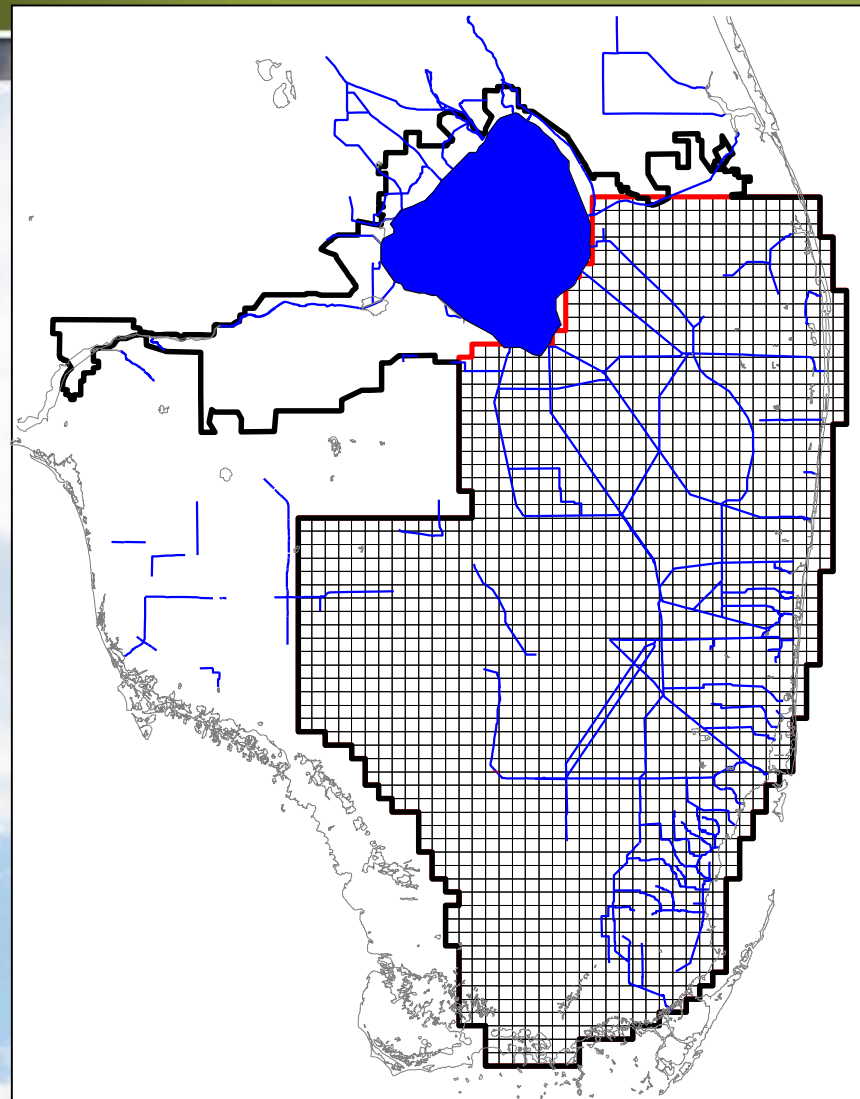
Hydrologic Performance Measures

www.sfwmd.gov/org/pld/restudy/hpm
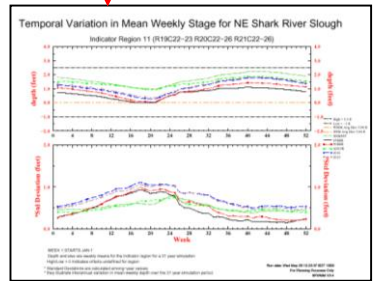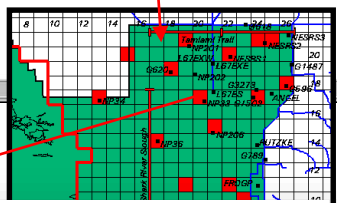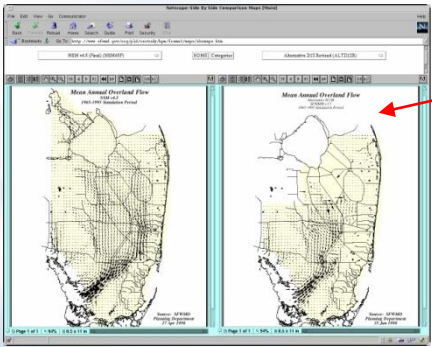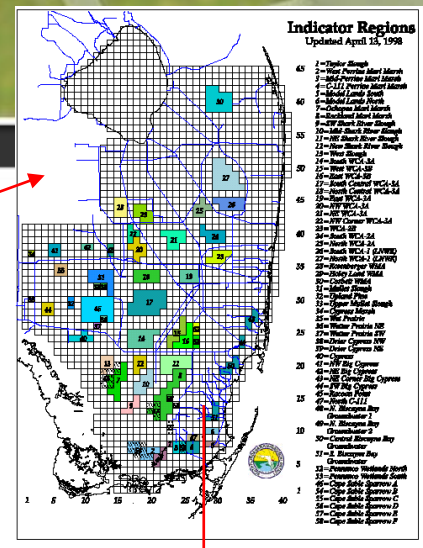
Dynamic Model for Stormwater Treatment Areas

RSM Engines

RSM

H S E
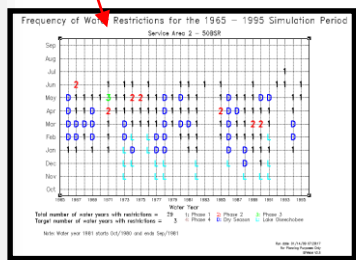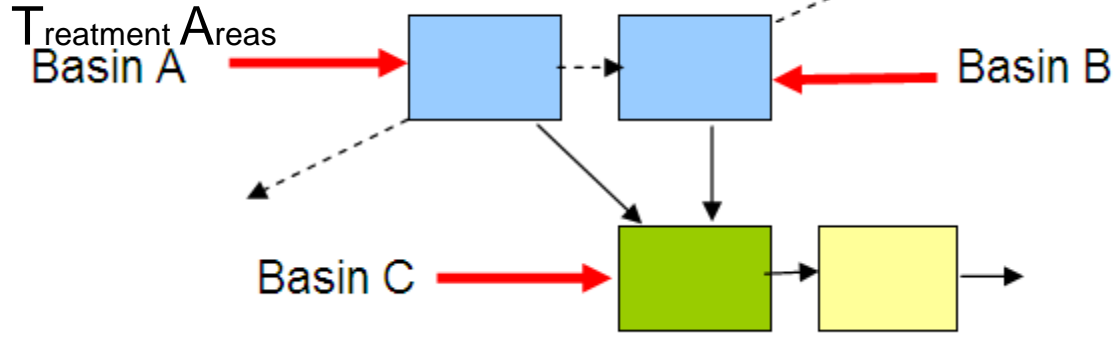
M S E

Hydrologic Simulation Engine (HSE)

- Model physical setup
- Simulate hydrologic processes
- Overland flow
- Groundwater flow
- Canal network
- Calibration/validation of model parameters
- Use observed structure flows
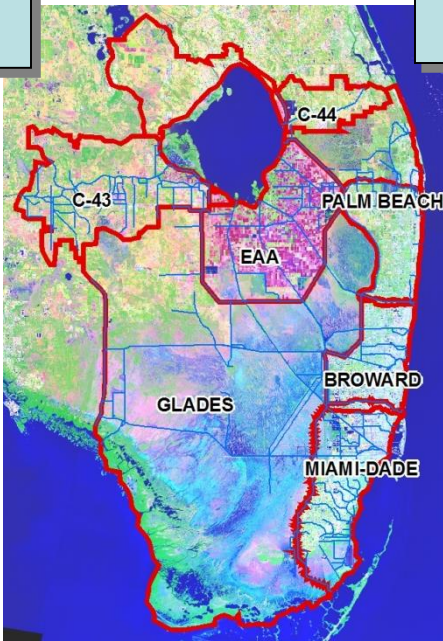
Management Simulation Engine (MSE)

- Simulate structure operations
- Implementation of operational rules
- Flood control rules
- Water supply policies
- Maintain minimum flows & levels
- Regional operational coordination

Numerical Mesh

- 5,794 triangular cells

- Mean & standard deviation of mesh cell sizes: 1.01 mi² & 0.74 mi²

- Mesh cell size range: 0.05 mi² to 3.92 mi²

- WCA-3B has the finest resolution; BCNP has the coarsest resolution

- WCA-3A has a total of 984 cells

- Average cell size in WCA-3A is 0.79 mi²; standard deviation is 0.24 mi²

Legend
- Structure
- Canals
- Basins
- Mesh
- Model Boundary

0   9   18   27   36
Miles

N
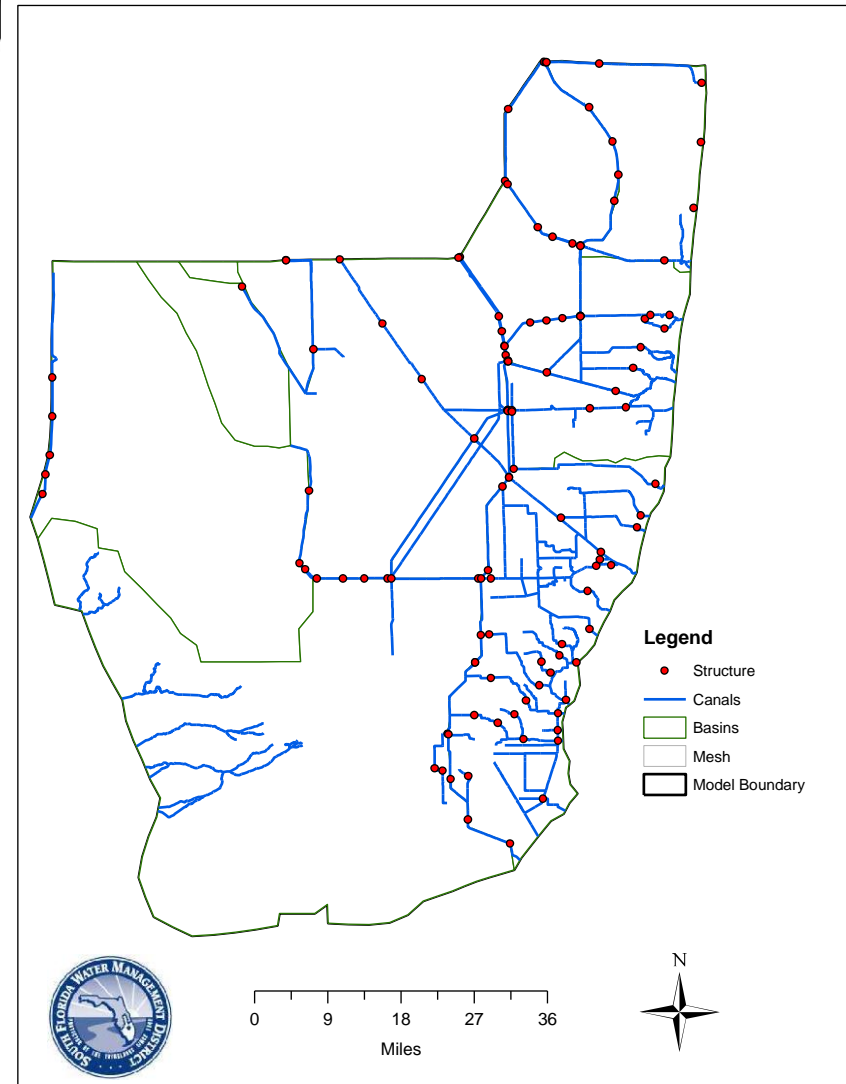
Previous Workshops at SFWMD

- <u>January 18-19, 1994</u> Workshop on Reduction of Uncertainties in Regional Hydrologic Simulation Models produced a report:
  - Sensitivity and Uncertainty Analysis in Hydrologic Simulation Modeling of the South Florida Water Management District Daniel P. Loucks and Jery R. Stedinger March 1, 1994

- <u>August 1995</u>: An evaluation of the certainty of system performance measures generated by the South Florida Water Management Model Paul J. Trimble.

- January 15-17, 2002:   MODEL UNCERTAINTY WORKSHOP produced a report
  - Quantifying and Communicating Model Uncertainty for Decision Making in the Everglades Upmanu Lall, Donald L. Phillips, Kenneth H. Reckhow and Daniel P. Loucks May 2002

- <u>September 24, 2004</u>: Uncertainty Workshop, Interagency Modeling Center : Presented by Christine Shoemaker, Jack Gwo and Wasantha Lal

- <u>May 2005</u>: Interagency Modeling Center Calculating MODFLOW Analytical Sensitivities Using ADIFOR for Effective and Efficient Estimation of Uncertainties Amir Gamliel, Mike Fagan and Maged Hussein

- <u>August 2005</u>: Interagency Modeling Center : Uncertainty of A Remediation Cost: A Demonstration of the NLH Technique in the analysis of uncertainty of objective value in model application Jack Gwo and George Shih

Bias, Precision, and Total Error

Total Error

Bias Error

Precision
Error

H_{True}

$H_{simulated}$
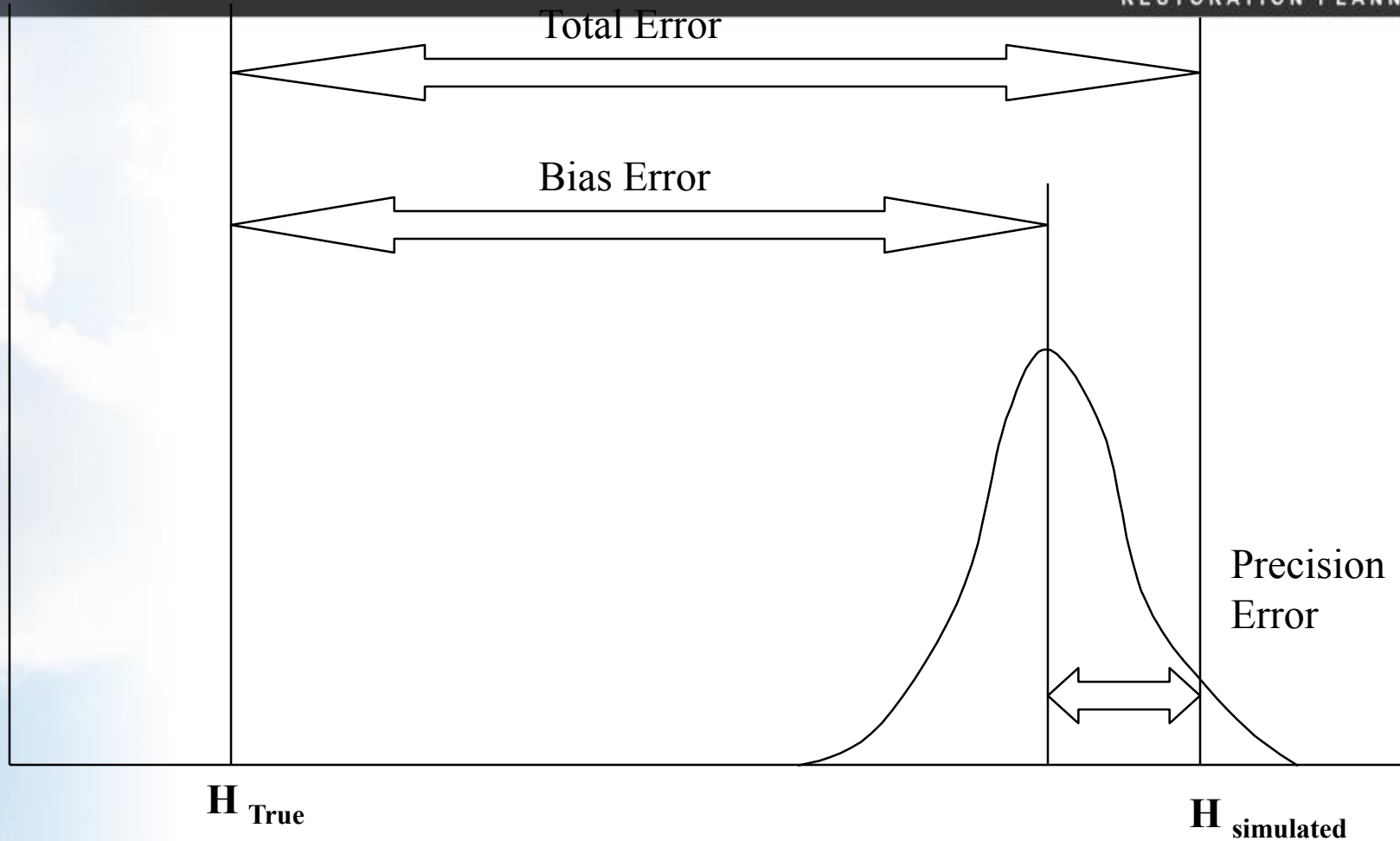
Uncertainty Analysis (UA)

- It determines the probability distribution of entire set of possible outcomes by considering the uncertainties in model input, parameters and algorithm.

- As it pertains to SFWMM/RSM, UA is a procedures of mapping uncertainty bands of model input/parameters/structure to uncertainty bands of model outcomes (prediction).

Uncertainty Analysis (UA)
Sensitivity Analysis (SA), definition

- A procedure to determine the sensitivity of model outcomes to changes in its parameters. If a small change in a parameter results in relatively large changes in the outcomes, the outcomes are said to be sensitive to that parameter.

Uncertainty Analysis (UA)

Sensitivity Analysis (SA), purpose

To understand which parameters are most critical for the model output

- To estimate parameter maximum and minimum values that provide <u>plausible</u> model outcomes for the purpose of providing some information about the parameter uncertainty.

- To calculate sensitivity matrix (Jacobian) which is a requirement for uncertainty analysis techniques.

Uncertainty Analysis (UA)

Importance Analysis (IA), OR Global Sensitivity Analysis

- Is the investigation of the combined effect of input uncertainty and the input/output sensitivity on the output uncertainty.

- Is the Isolation of the input parameters with most contribution to model output variance.

- Function of input uncertainty and output sensitivity to that input

- IA techniques:

  - Stepwise Rank Regression Analysis

  - Classification Tree Analysis

Uncertainty Quantification

Why is it so difficult ?

- Input variables, such as rainfall, ET, Landuse, ..etc., contain stochastic components and are pre-processed based on other models (physically or statistically based).

- Model parameters are highly random and may change spatially and/or seasonally.

- Model formulation and parameterization are complex processes

- System Compartmentalization, and System Management and operation add more dimensions to the already complex system hydrology.

- With 500+ variables in such environment, Uncertainty Analysis is a challenge.

Sources of Uncertainty

- Uncertainty due to our inability to fully understand the natural variability of input process to the model at a scale smaller than the gauging scale. Examples of these uncertainties are:

  - Spatial variability such as rainfall, PET, and topography

  - Temporal variability such as inflow and tidal boundary conditions

SOURCES OF UNCERTAINTY, cont.

- Uncertainty due to measurement errors. This covers all field measurements and published data based on which input and output data are directly used, or estimated using an external data processing (or modeling).

- Uncertainty due to conceptual and implementation errors :-

  - Error in specifying boundary conditions such as inflow and tidal boundaries and initial conditions such as stage.

  - Model structural and numerical errors

SOURCES OF UNCERTAINTY, cont.

- Conceptual and implementation errors (cont.)

  - Model parameter errors due to parameter modeling errors and/or calibration imperfection.

  - Model inability to resolve variability smaller than the designated time step and mesh cell size

  - Temporal and spatial discretizations and their interdependence

- Model linkage to other models

  - Water quality and hydrologic model integration/coupling

  - Input preprocessing models (demands, runoffs, rainfall, ..etc.

MEASURES AND SOME USES OF UNCERTAINTY

- In its simple format, a mean and a standard deviation of a given output, performance measure or index. This simplified uncertainty metric is rarely sufficient for a complete characterization of uncertainty.

- Model output in terms of a range rather than a single value. This describes the system performance as a range of potential outputs, classes of likely events, or probability density function.

- Provides a level of confidence that a certain output is within an acceptable performance indicators.

- Provides probability that a certain output exceeds a specific target value.

TECHNIQUES TO QUANTIFY UNCERTAINTY

- ANALYTICAL:

  - Derive the output error distribution (e.g., variance)

  - Feasible for simple models with few stochastic (random) input parameters.

  - Given the complexities and large variables in our models, this approach does not go very far.

TECHNIQUES TO QUANTIFY UNCERTAINTY

- **FIRST-ORDER SECOND MOMENT ANALYSES:**

  - This method derives the output variance from input parameter variance / covariance functions

  - This method can identify the relative contribution of each parameter to the output variance.

  - Suitable when parameter-output relationship is linear or mildly nonlinear.

  - If the linearity condition is not "properly" satisfied, then second order term of Taylor expansion must be considered and a correction term must be applied

  - Refer to Loucks & Stedinger 1994, Trimble 1995, and Lal 1995.

TECHNIQUES TO QUANTIFY UNCERTAINTY

- **FIRST-ORDER SECOND MOMENT ANALYSES:**

$$F(\mathbf{x}) \cong F(\hat{\mathbf{x}}) + \sum_i \frac{\partial F}{\partial x_i}(x_i - \hat{x}_i)$$

$$V[F] = \sigma_F^2 = \mathbf{E}[(F - \mathbf{E}[F])^2]$$

$$V(F) \cong \sum_i \sum_j \frac{\partial F}{\partial x_i}\frac{\partial F}{\partial x_j}\mathbf{E}[(x_i - \hat{x}_i)(x_j - \hat{x}_j)] \cong \sum_i \sum_j \frac{\partial F}{\partial x_i}\frac{\partial F}{\partial x_j}C[x_i x_j]$$

$$V(F) \cong \sum_i \left(\frac{\partial F}{\partial x_i}\right)^2 V[x_i]$$

TECHNIQUES TO QUANTIFY UNCERTAINTY

- Stochastic Numerical Models:

  - Develop and solve the governing equation with stochastic component

  - Probability distribution is inherent in the solution

  - Very simple models compared to SFWMD system

  - Numerical solution of such a stochastic equation is far more complex than the already challenging solution of the deterministic equation.

TECHNIQUES TO QUANTIFY UNCERTAINTY

- **Monte Carlo with Random Sampling**

  - Recognize some input variables/parameters as random. Identify their probability distributions by expert judgment and historical data.

  - For each simulation model run, draw the actual values of input variables/parameters from their respective distribution. Record the corresponding output.

TECHNIQUES TO QUANTIFY UNCERTAINTY

- Monte Carlo with Random Sampling (cont.)

  - With considerable number of simulations and many recorded outputs (all are equally likely outcomes), obtain output probability distribution.

  - Massive number of simulations is needed

  - Input parameters/variables are likely correlated both in space and time and hence sampling must be drawn from a joint probability distribution that reflect both scales. The construction of such distributions is not easy

TECHNIQUES TO QUANTIFY UNCERTAINTY

- Bayes' Theorm

$$P(K \mid Q) = \frac{P(Q \mid K)P(K)}{P(Q)}$$

P(*K*) is the prior (marginal) probability of *K (e.g., Hydraulic Conductivity)*.
P(*K*|*Q*) is the conditional (posterior) probability of *K*, given *Q (e.g., Observed flow)*.
P(*Q*|*K*) is the conditional probability of *Q* given *K*. It is also called the likelihood of K for observed Q. → P(*Q*|*K*) ≈ L(*K*|*Q*). A measure of the ability of "K" set in predicting the Observed "Q" set.
P(*Q*) is the prior or marginal probability of *Q*, and acts as a normalizing constant.
Bayes' theorem in this form gives a mathematical representation of how the conditional probability of event *K* given *Q* is related to the converse conditional probability of *Q* given *K*.

TECHNIQUES TO QUANTIFY UNCERTAINTY

- Bayesian Monte Carlo analysis

  - Combine prior information about the input parameter distribution with the ability of these parameters to describe available data on state variables.

  - Start with the traditional Monte Carlo sampling from prior distributions.

  - Compare each simulation results to field observations of the model state variables (e.g., flow) and Score each results with respect to the ability of each parameter set to describe the observed data.

TECHNIQUES TO QUANTIFY UNCERTAINTY

- Bayesian Monte Carlo analysis

  - Scoring system can be as simple as yes/no binary function or it can be based on a likelihood function $P(Q|K) \approx L(K|Q)$.

  - Perform sufficient simulations for the n parameters and build n-dimensional matrix describing the marginal parameter uncertainty and the entire error covariance structure.

  - You can do one of two things: define model prediction uncertainty or investigate the parameter individual contributions to overall uncertainty.

TECHNIQUES TO QUANTIFY UNCERTAINTY

- **G**eneralized **L**ikelihood **U**ncertainty **E**stimation

- **M**onte **C**arlo **M**arkov **C**hain
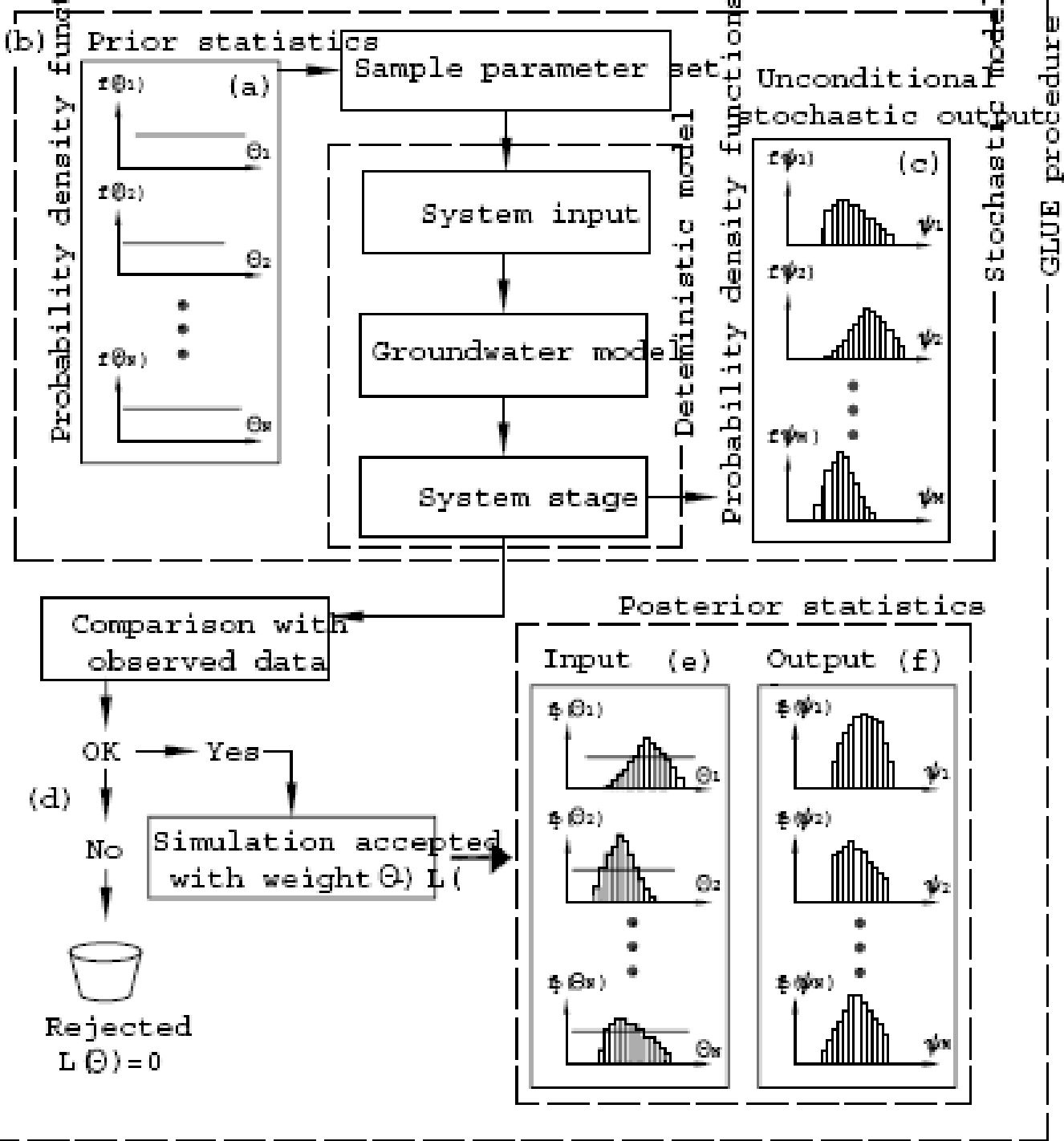
DISADVANTAGE OF MONTE CARLO TECHNIQUE

- Large number of simulations is expensive computationally especially for distributed models with long run time

- Risk of obtaining unrealistic combinations of input values especially if the input variables/parameters are NOT independent.

TECHNIQUES FOR COMPUTATION EFFICIENCY

- **Latin Hypercube Sampling**

  - It reduces the number of input sampling variability

  - For each input variable/parameter, the probability distribution is divided into segments of equal probability

  - The algorithm assures sampling only once from each segment.

  - Modification to this algorithm considers the variables/parameters interdependency

Latin hypercube sampling method
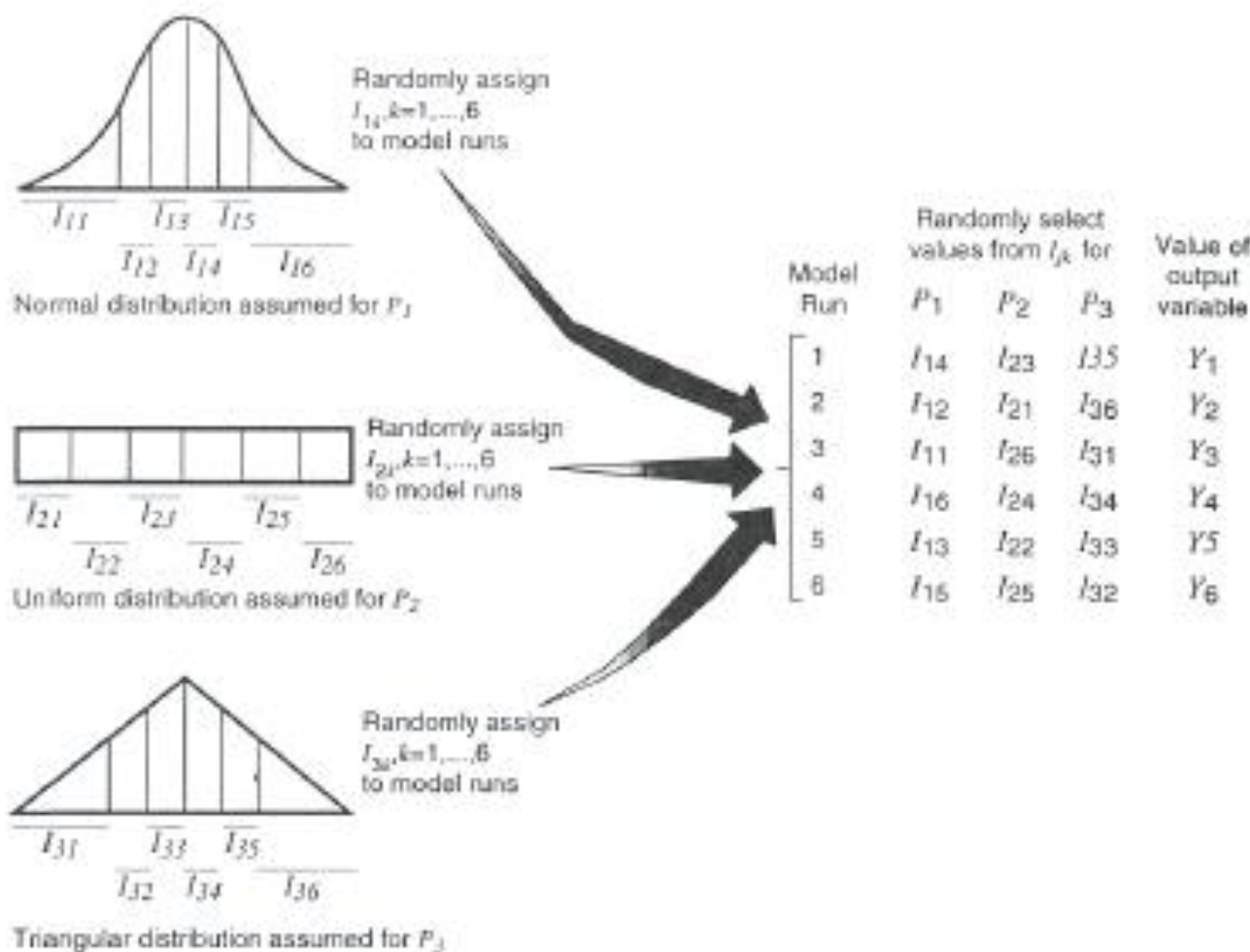
Fig. 29: Schematic representation of a Latin Hypercube sampling study of six runs.

NSRSM UNCERTAINTY ANALYSIS OBJECTIVES **

- Considering a select group of parameters:

  - Provide local sensitivity analysis

  - Provide uncertainty analysis using more than one technique.

  - Provide Global sensitivity analysis

STEPS**

1. Selection of a limited set of key inputs and outputs based on previous modeling studies and expert opinion.

2. Application of formal local sensitivity analysis (via Singular Value Decomposition of the input-output sensitivity matrix) to identify significant input uncertainties.

3. Assignment of probability distributions to characterize uncertainty in selected model inputs and their correlation structure (based on the best available data)

STEPS (cont.)**

4. Application of uncertainty quantification techniques to determine the uncertainty in model output (s) as a function of the uncertainty in model inputs.

5. Application of global sensitivity (uncertainty importance) analysis techniques to identify those model inputs that are key contributors to the overall uncertainty in model output(s). This results in an importance ranking that is dependent on both input uncertainty and input-output sensitivity, whereas the importance ranking based on SVD factorization is only dependent on input-output sensitivity.

1) Input/Output Selection

Legend:
- NSRSM Model Boundary
- Ridge and Slough Marsh
- Mesic Pine Flatwood
- Rivers

Map labels: T712_East Transect, 25087, 25492, Tamiami Transect

Scale: 0 5 10 20 30 Miles
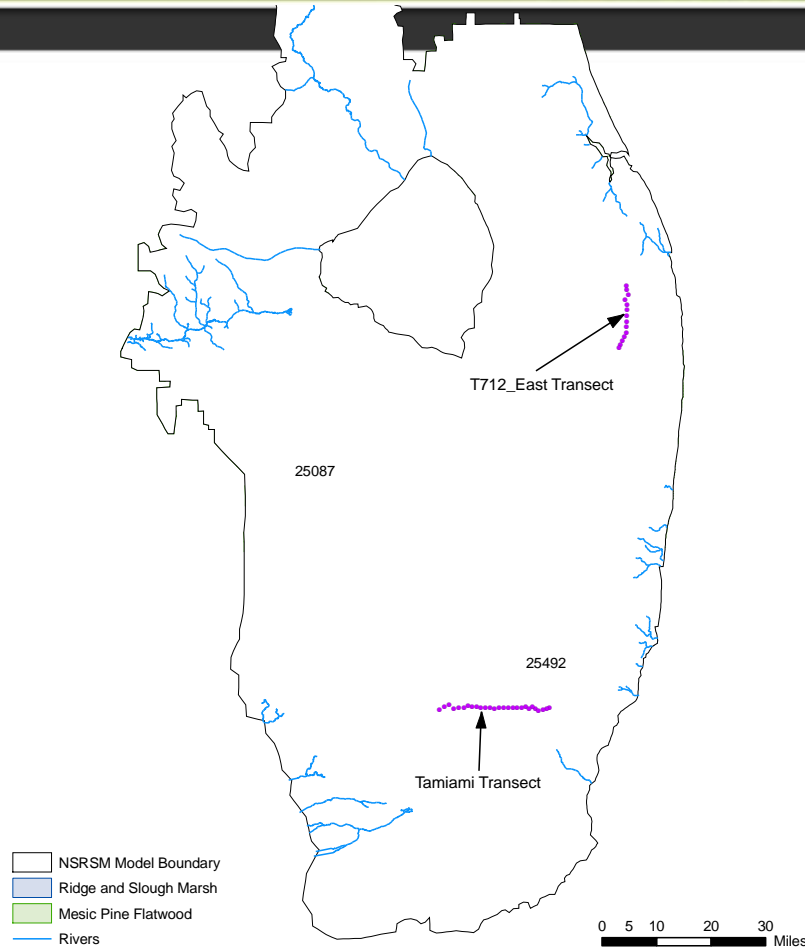
Land cover types chosen for parameter variation and locations of output metrics.

1) Input/Output Selection : Model Parameters**

Parameter	Description	Data range in NSRSM	Ridge and Slough Marsh		Mesic Pine Flatwood	
			Original Value	Abbrevia-tion	Original Value	Abbrevia-tion
Alpha	Conveyance – *a* parameter	0.1 - 0.4 mostly 0.3	0.325	alph511	0.3	alpha712
Detent	Conveyance – detention storage	0.1	0.1	detent511	0.1	detent712
Xd	ET – extinction depth	3-10	3	xd51	10	xd712
Kveg	ET – kveg	-0.1 - 1.0	0.74	kveg511	0.74	kveg712
Storativity	Hydrogeology – specific yield	.2 – 1.0 mostly .2	0.8	sv511	0.2	sv712

1) Input/Output Selection : Output Metric**

Metric Type	Location	Abbreviation
Stage	25492, land cover 511	25492stage
Stage	25087, land cover 712	25087stage
Transect Flow	land cover 511	Tamiami
Transect Flow	land cover 712	T712_East

2) SVD-Based Local Sensitivity Analysis:

SVD →Singular Value Decomposition[**]

- Consider Sensitivity Matrix (Jacobian) A_{mXn} , with an entry $\alpha_{i,j}$

$$\alpha_{i,j} = \frac{\partial h_j}{\partial k_i} ....i = 1,2,...,n........j = 1,2......,m$$

- α_{ij} = the sensitivity of the jth simulated output metric to the ith parameter

- h_j = the jth simulated output

- k_i = the ith parameter

- m = # of observations, n = # of parameters

Matrix A can be decomposed into three matrices V, S, and U

$$A = U_{mxm} . S_{mxn} . V_{nxn}^{T}$$

- **S** is a diagonal matrix of singular values of *A (i.e., the value that makes the corresponding row of matrix A = 0.)*

- V^T gives the coefficients of linear combinations of the original parameters that give rise to new, independent parameter groups

- *U* gives the coefficients of linear combinations of the observation groups.

- The parameter groups and observation groups are related by the diagonal matrix **S**

- The relative magnitude of the singular values in *S* indicates the relative importance of each of the parameter groups

Other Important Matrices for Sensitivity Analysis

- *Resolution Matrix gives insight regarding parameter resolution (parameter interdependence)*

$$R = V.V^T$$

- *Correlation Matrix gives insight regarding parameter resolution (parameter interdependence)*

$$\Sigma = V(1/S^2).V^T ,.. \rho_{i,j} = \frac{\sigma^2_{ij}}{\sigma_{ii}.\sigma_{jj}}$$

- **The singular values, *U* and *V^T*, the resolution matrix, and the correlation matrix are the primary sources of information used to construct groups of parameters, understand their interdependence, and analyze their sensitivity**

RESULTS: SVD-BASED SENSITIVITY ANALYSIS
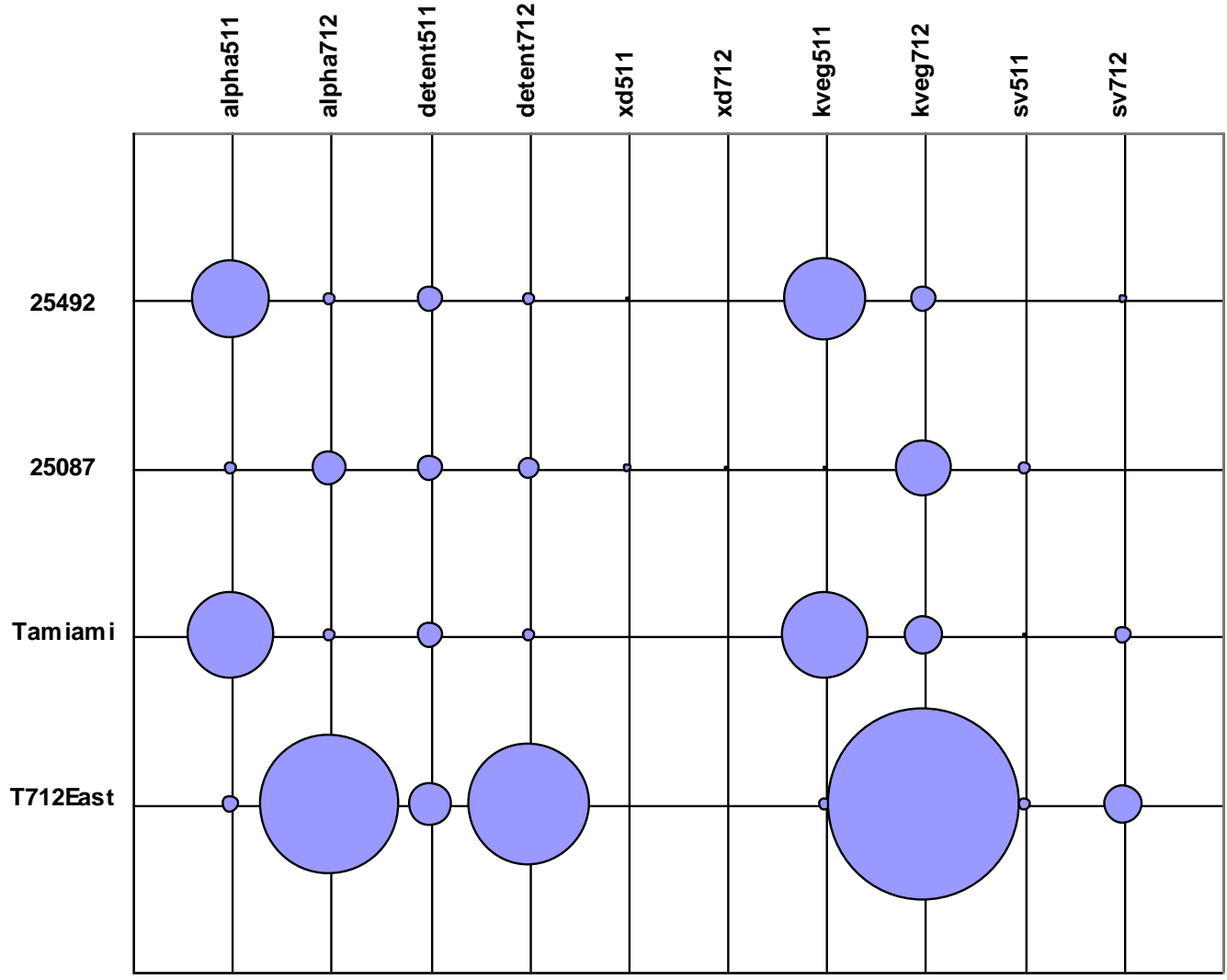
Bubble plot of the sensitivity matrix

RESULTS: SVD-BASED SENSITIVITY ANALYSIS
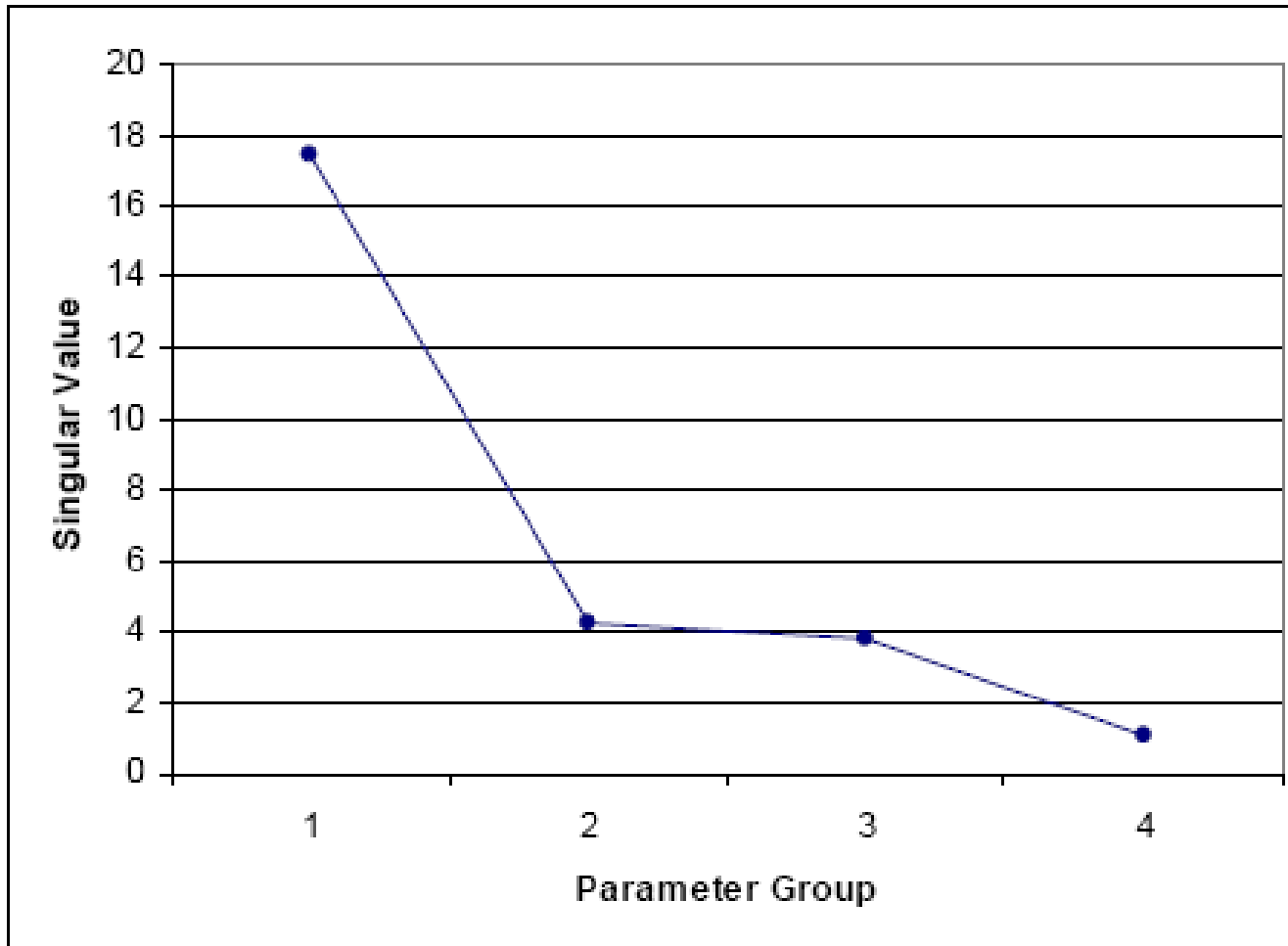
Singular values from the SVD decomposition,
Cutoff to control data error: $s_{min}/s_{max} < 0.001$

RESULTS: SVD-BASED SENSITIVITY ANALYSIS

STORATION PLANNING
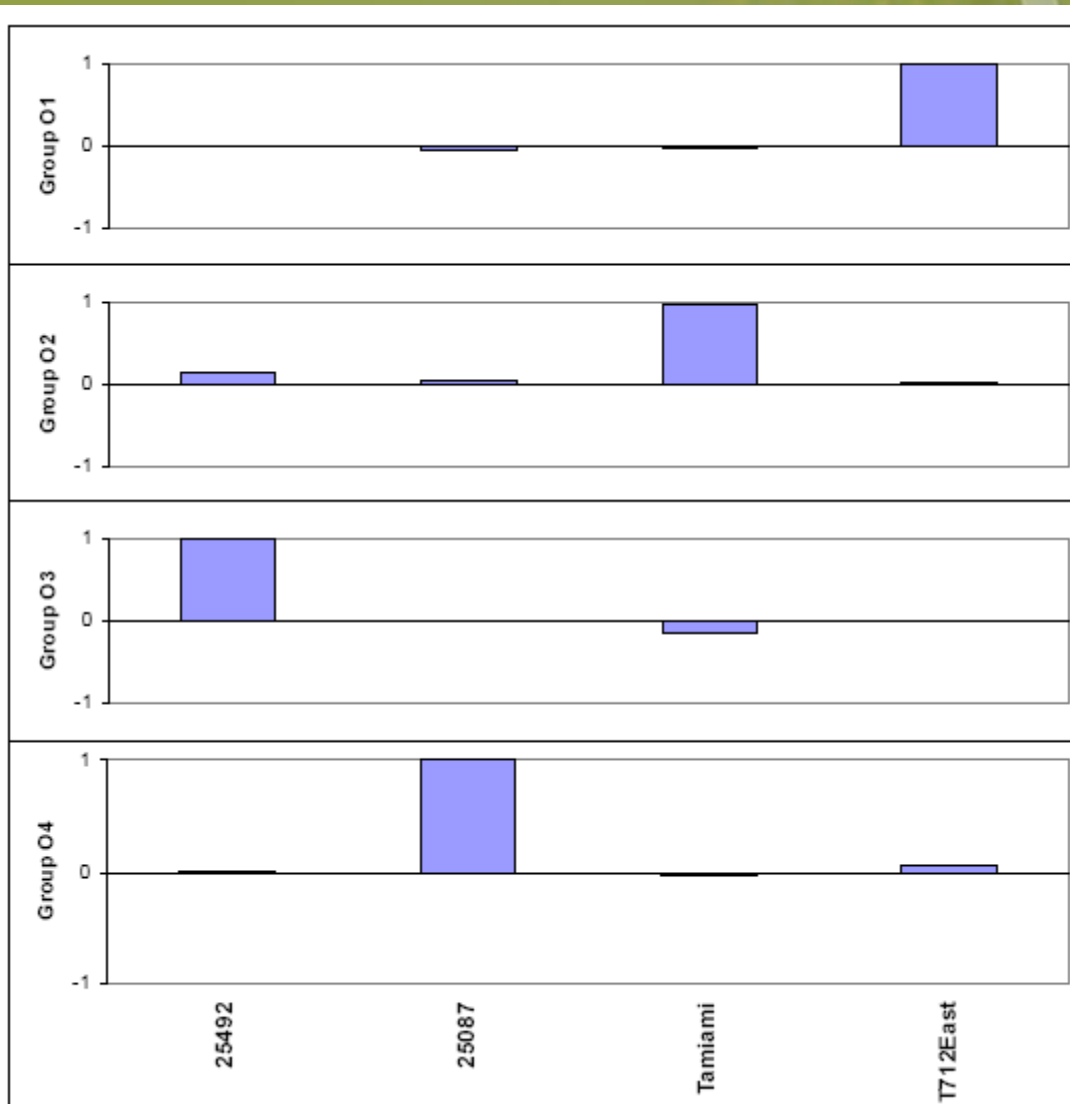
U matrix elements showing linear coefficients of the output groups

RESULTS: SVD-BASED SENSITIVITY ANALYSIS

STORATION PLANNING

Elements of the V^T matrix showing linear coefficients of parameter groups

RESULTS: SVD-BASED SENSITIVITY ANALYSIS

Bubble plot of the Resolution matrix
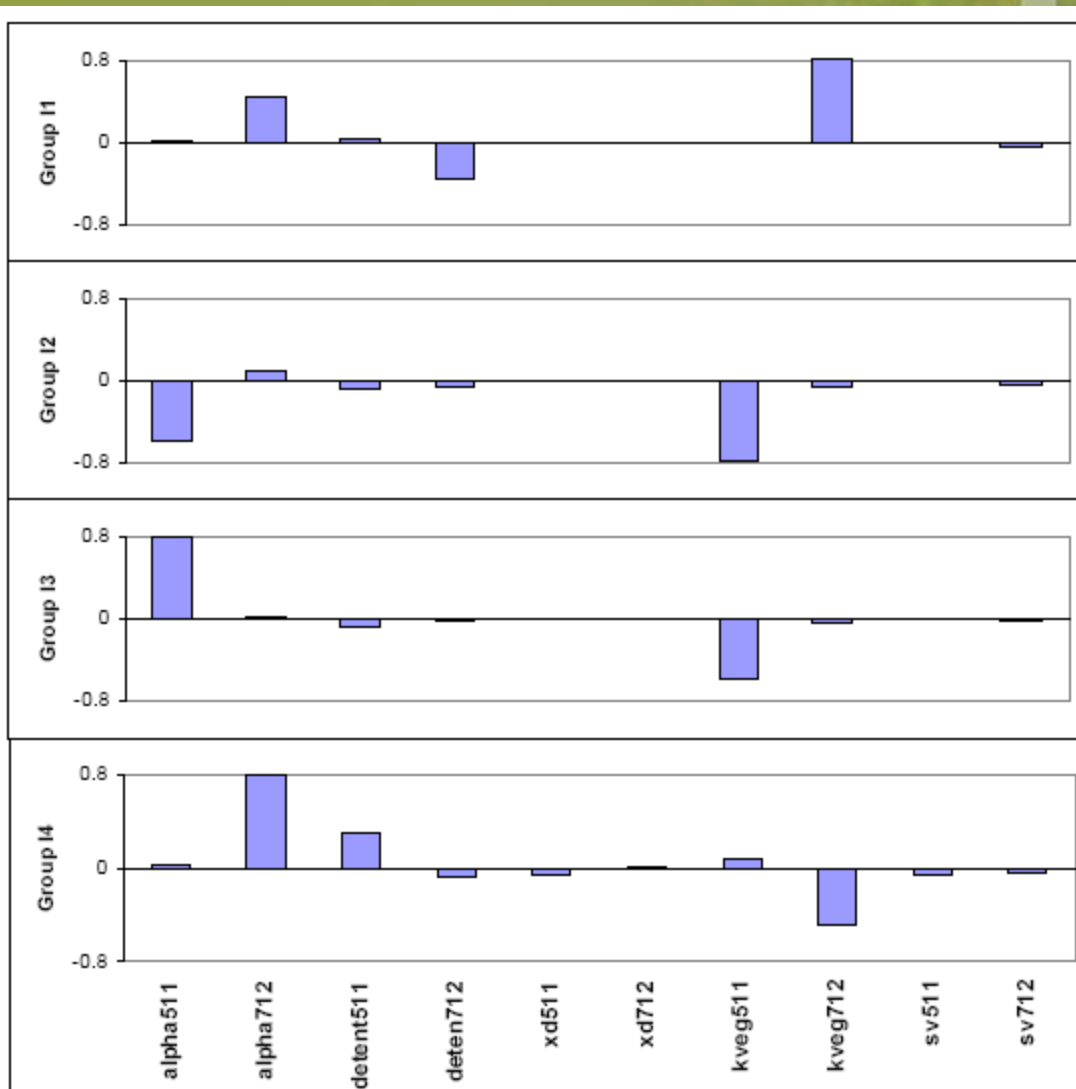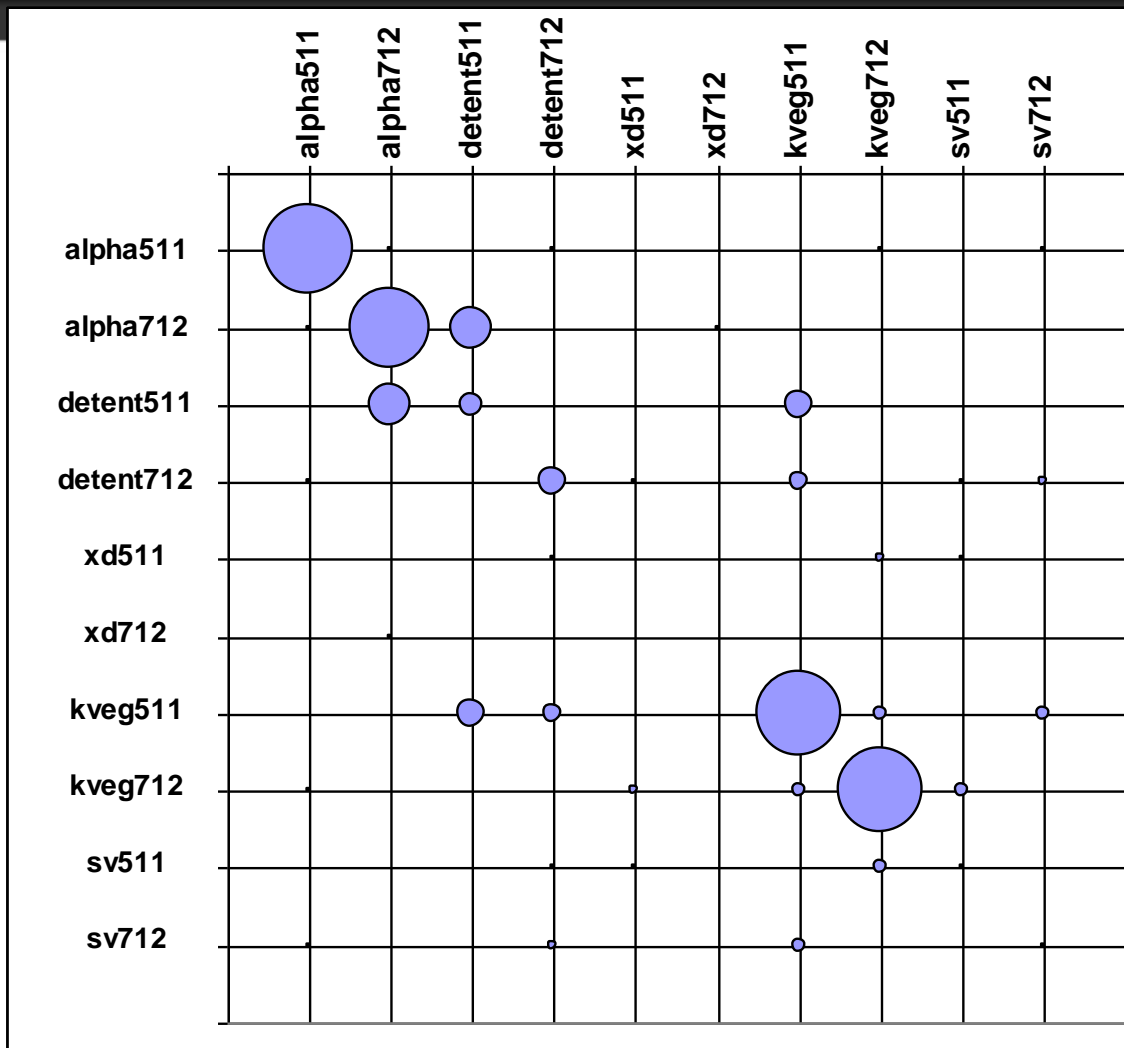
RESULTS: SVD-BASED SENSITIVITY ANALYSIS
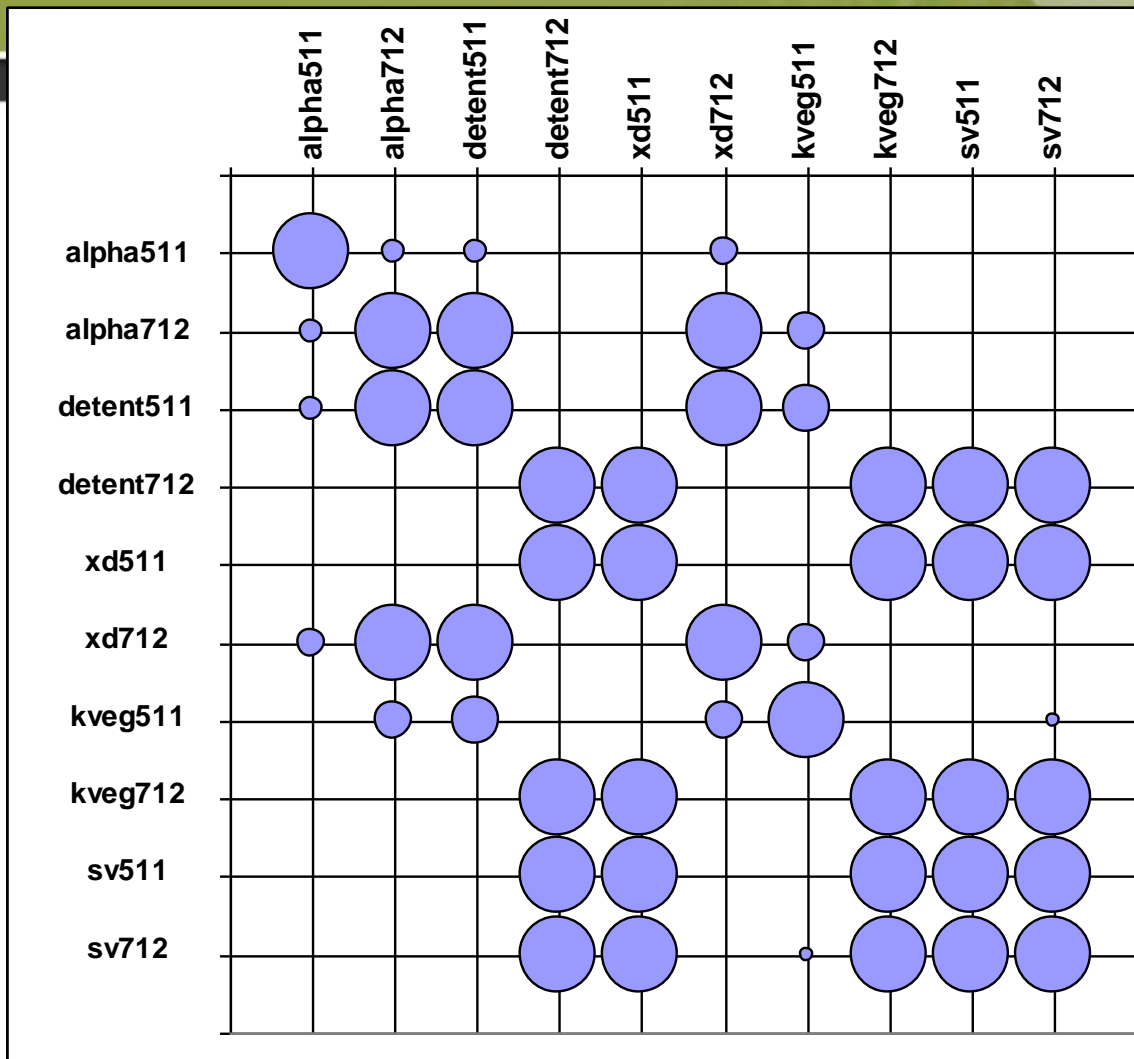
Bubble plot of the Correlation matrix

3) CHARACTERIZATION OF PARAMETER UNCERTAINTY

Conveyance

Ridge and Slough Marsh (.325)		Mesic Pine Flatwood (.3)	
Value	CDF	Value	CDF
.06	0	.3	0
.3	.15	.35	.10
.35	.95	.45	.90
.4	1	.6	1

Manning's *n*

Ridge and Slough Marsh (.1)		Mesic Pine Flatwood (.1)	
Value	CDF	Value	CDF
.1	0	.1	0
.6	1	.2	1

Detention Storage

ET

Ridge and Slough Marsh (.88)		Mesic Pine Flatwood (.84)	
Value	CDF	Value	CDF
.7	0	.4	0
.8	.5	.6	.40
.9	1.0	.7	.90
		.8	1

Vegetation Crop Coefficient

Ridge and Slough Marsh (3.0)	Mesic Pine Flatwood (10.0)
2-4 Normal Distribution 3.0 mean .33 standard dev.	8-12 Normal Distribution 10.0 mean 0.667 stand dev

Extinction Depth

CHARACTERIZATION OF PARAMETER UNCERTAINTY

Ridge and Slough Marsh (.8 with lookup)		Mesic Pine Flatwood (.2)
Value	**CDF**	.1-.3 Normal Distribution .2 mean .033 stand dev
.5	0	
.6	.25	
.7	.50	
.8	1.0	

Storage Coefficient

4) UNCERTAINTY Quantification:

- Monte Carlo Simulation

- First Order Second Moment Analysis

UNCERTAINTY PROPAGATION **:
Comparison of MCS and FOSM results.

	FOSM		MCS	
	Mean	**Stdev**	**Mean**	**Stdev**
25492stage	1.26	0.12	1.26	0.14
24087stage	0.13	0.054	0.13	0.047
Tamiami	2.55E+08	3.25E+07	2.70E+08	7.86E+07
T712_East	-8.49E+07	9.80E+06	-8.25E+07	1.13E+07

UNCERTAINTY IMPORTANCE ANALYSIS: Global Sensitivity **

- Is the investigation of the combined effect of input uncertainty and the input/output sensitivity on the output uncertainty.

- Is the Isolation of the input parameters with most contribution to model output variance.

- Two techniques are employed:
  - Stepwise Rank Regression Analysis
  - Classification Tree Analysis

UNCERTAINTY IMPORTANCE ANALYSIS: Stepwise Rank Regression Analysis

- Fit a linear response surface between the rank-transformed input and output variables and Perform a sensitivity analysis on this "surrogate" model.

- Include variables to the regression in a stepwise fashion. The order by which variables are added to the regression model corresponds to their order of importance.

- The order of importance is measured by the relative contribution to the regression variance.

- The stepwise regression process continues until the input-output model contains all of the input variables that explain "statistically significant" amounts of variance

Stepwise Rank Regression Analysis

Stepwise-Regression Analysis Results for metric [25492stage].

Rank	Variable	R^2	SRC
1	KVEG511	0.379	-0.632
2	ALPHA511	0.545	0.425
3	TOPOSELECT	0.653	0.322
4	DETENT511	0.750	0.315
5	KVEG712	0.819	-0.264

Stepwise-Regression Analysis Results for metric [25087stage].

Rank	Variable	R^2	SRC
1	KVEG712	0.971	-0.981
2	ALPHA712	0.981	0.099
3	DETENT712	0.982	0.044

Stepwise-Regression Analysis Results for metric [Tamiami].

Rank	Variable	R^2	SRC
1	KVEG511	0.557	-0.679
2	ALPHA511	0.742	-0.428
3	KVEG712	0.810	-0.259
4	DETENT511	0.861	0.214
5	XD511	0.871	0.103

Stepwise-Regression Analysis Results for metric [T712_East].

Rank	Variable	R^2	SRC
1	KVEG712	0.620	0.800
2	ALPHA712	0.889	0.508
3	DETENT712	0.941	-0.228
4	ALPHA511	0.943	0.048

UNCERTAINTY IMPORTANCE ANALYSIS:
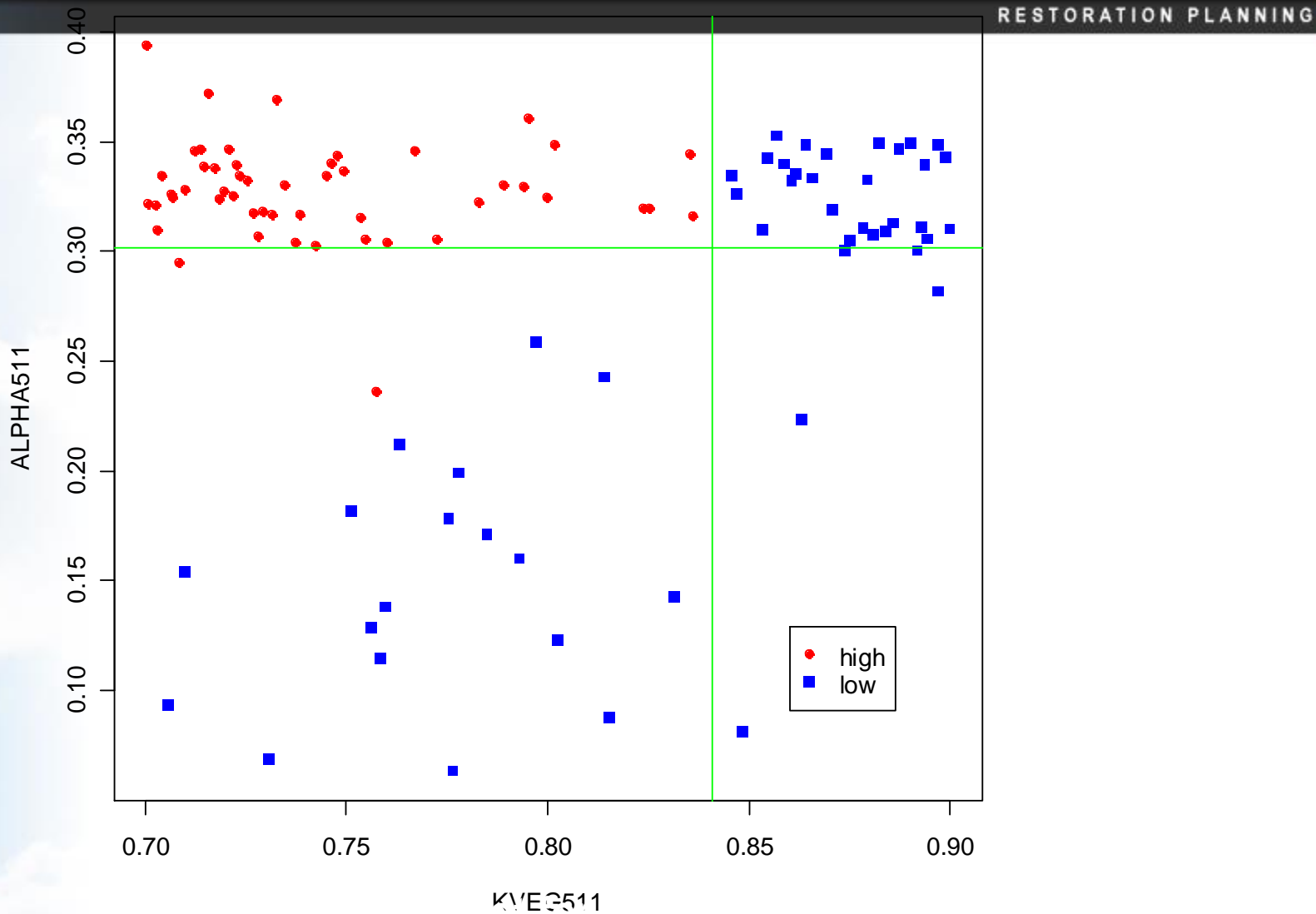## Classification Tree Analysis

- The decision tree is generated by recursively finding the variable splits that best separate the output into groups where a single category dominates.

- The importance of the variables is demonstrated by their order of split, with the variables at the top of the classification tree (the first variables split) considered more important than the variables involved in later splits

Classification Tree Analysis

KVEG511< 0.8409

ALPHA511>=0.2766

low
0/32

high
49/0

low
1/18
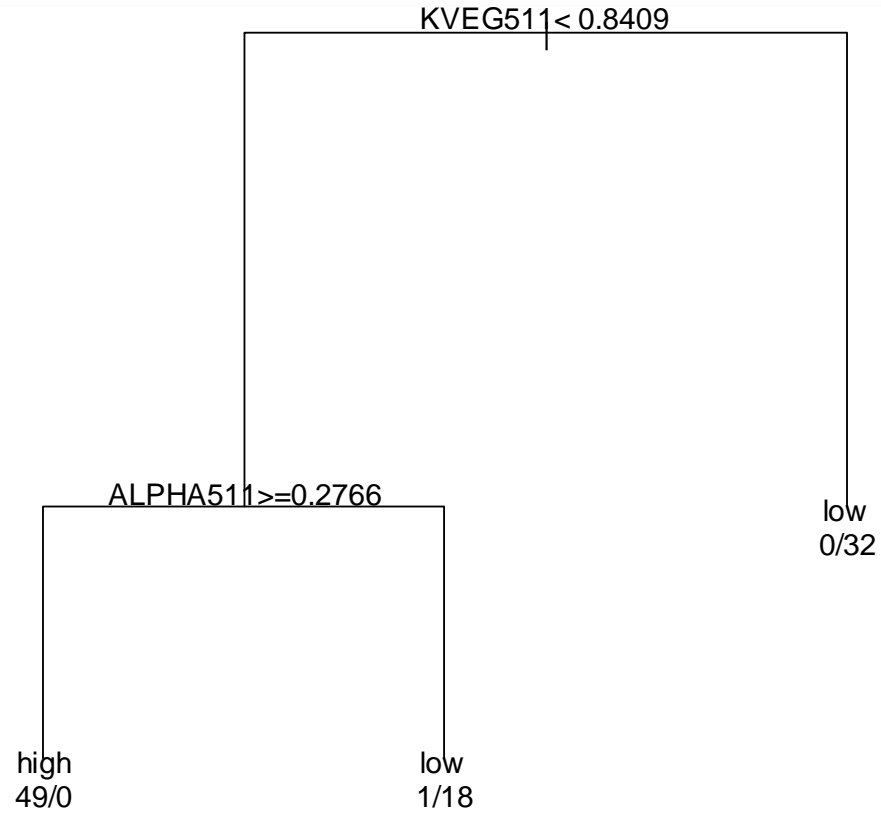
Classification Tree Analysis

CONCLUSION **

- SVD, Stepwise rank regression and classification tree analysis are useful tools in isolating and identifying parameters contributing to model output sensitivity and uncertainty.

- Monte Carlo Simulation is a powerful (but expensive) tool for full characterization of model output uncertainty.

- FOSM analysis can be a useful tool in lieu of MCS provided that 1) Gaussian and stationarity assumptions are reasonably satisfied, and 2) mean and variance are the user's primary interests.

- Among the parameters considered, Crop Coefficient Kveg, and (Manning Conveyance Alpha with lesser extent) have the greatest contribution to model output uncertainty.

CONCLUSION **

- The uncertainty analysis was "sensitive" to the location of the time slice selected.

- CDFs obtained at various time slices exhibited non-stationarity that must be addressed and must be linked to the subsequent use of the uncertainty analysis.

DISTRICT LONG TERM GOAL FOR UNCERTAINTY ANALYSIS **

- Identify, isolate, and quantify those sources of uncertainties with significant and unique contribution to the overall model output uncertainty.

- Develop a suite of Uncertainty and Sensitivity Analysis tools for all the district hydrologic models.

- Provide the enduser with a decision making tool that enables him/her infer the model output uncertainty given all variables and parameters presented above.

- Identify areas of improvement in all sources of uncertainties identified above.

Lessons learned

- UA is a long term journey that needs to be harbored in house.

- In house staff to lay out short and long term plans for uncertainty analysis.

- Pursue UA short term goals for model "endorsement", for proof of concepts, pilot studies, …etc.

- Pursue UA long term goals

  - Develop simpler (more parsimonious) models in consistency with the available data.

  - Manage performance measures and enduser expectations.

Lessons learned

- Pursue more comprehensive UA (beyond parameterization) including other factors such as input data, boundary conditions, management rules, …etc.

- Initiate a data collection program to allow for real time analysis and model updating → reduce uncertainty

- Pursue Bayesian Networks and Bayesian Approach to combine priori and posterior information to improve prediction.

- Don't be "married" to one school of thought, to one type of expertise, or to one technique.

- The utilization of uncertainty results by the end user is yet another difficult task.